

On the breaking of a plasma wave in a thermal plasma: II. Electromagnetic wave interaction with the breaking plasma wave

Sergei V. Bulanov,^{1,*} Timur Zh. Esirkepov,¹ Masaki Kando,¹ James K. Koga,¹ Alexander S. Pirozhkov,¹ Tatsufumi Nakamura,¹ Stepan S. Bulanov,^{2,†} Carl B. Schroeder,³ Eric Esarey,³ Francesco Califano,⁴ and Francesco Pegoraro⁴

¹*QuBS, Japan Atomic Energy Agency, 1-8-7 Umemidai, Kizugawa, Kyoto, 619-0215 Japan*

²*University of California, Berkeley, CA 94720, USA*

³*Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

⁴*Physical Department, University of Pisa, Pisa 56127, Italy*

(Dated: 19/Apr/2012, 12:30, Japan time)

The structure of the density singularity formed in a relativistically large amplitude plasma wave close to the wavebreaking limit leads to a refraction coefficient which has a coordinate dependence with discontinuous derivatives. This results in a non-exponentially small above-barrier reflection of an electromagnetic wave interacting with the nonlinear plasma wave.

PACS numbers: 52.38.Ph, 52.35.Mw, 52.59.Ye

* Also at A. M. Prokhorov Institute of General Physics of RAS, Moscow, Russia

† Also at Institute of Theoretical and Experimental Physics, Moscow 117218, Russia

I. INTRODUCTION

In the first part of our paper [1], extending an approach formulated in Ref. [2] to the relativistic limit, we have studied systematically the structure of the singularities formed in a relativistically large amplitude plasma wave close to the wavebreaking in a thermal plasma. We have shown that typically the electron density distribution in the breaking wave has a “peakon” form with a discontinuous coordinate dependence of its first derivative, similar to the profiles of nonlinear water waves [3–5] and that in the above breaking limit the derivative becomes infinite. This results in a finite reflectivity of an electromagnetic wave interacting with nonlinear plasma waves. In particular, this is an important property because nonlinear Langmuir waves play a key role in the “relativistic flying mirror” concept [6–11]. In this concept, very high density electron shells are formed in the nonlinear wake wave generated by an ultrashort laser pulse propagating in an underdense plasma with a speed close to the speed of light in vacuum. The shells act as mirrors flying with relativistic velocity. When they reflect a counterpropagating electromagnetic pulse, the pulse is compressed, its frequency is upshifted and its intensity increased. It is the singularity in the electron density distribution that allows for a high efficiency in the reflection of a portion of the counterpropagating electromagnetic pulse. If the Langmuir wave is far below the wave-breaking threshold, its reflectivity is exponentially small. For a nonlinear Langmuir wave the singularity formed in the electron density breaks the geometric optics approximation and leads to a reflection coefficient that is not exponentially small [6, 12].

In the present paper we address the problem of the interaction of an electromagnetic wave with a nonlinear plasma wave which is of interest for the “photon accelerator” concept [14] and for the “relativistic flying mirror” paradigm [6–11]. We calculate the reflection coefficients of an electromagnetic wave at the singularities of the electron density in the most typical regimes of a strongly nonlinear wave breaking in thermal plasmas.

II. ELECTROMAGNETIC WAVE REFLECTION BY THE ELECTRON DENSITY MODULATED IN THE BREAKING WAVE

As we have seen in the first part of our paper [1], in a strongly nonlinear wake wave the electron density is modulated and forms thin shells (singularities or caustics in the plasma flow) moving with velocity β_{ph} . In the Introduction, in a way of Refs. [6–11], we have discussed how a counterpropagating electromagnetic wave can be partially reflected from these density shells which play the role of relativistic mirrors. While in the case of a cold plasma the electron density at the singularity tends to infinity (see Eq. (59) of Part I [1] and Refs. [7, 12]), in a thermal plasma the density is limited by the expressions given by Eqs. (41) and (42) of Part I [1]. Although in this case the density profile is described by a continuous function of the variable X , its derivatives with respect to X are discontinuous. This discontinuity results in the breaking of the geometric optics approximation and leads to a reflectivity that is not exponentially small.

In order to calculate the reflection coefficient, we consider the interaction of an electromagnetic wave with the electron density shell formed at the breaking point of a Langmuir wave in a thermal plasma similarly to what has been done in Refs. [6, 12]. The electromagnetic wave, described by the z component of the vector potential $A_z(x, y, t)$, evolves according to the linearized wave equation

$$\partial_{tt}A_z - c^2(\partial_{xx}A_z + \partial_{yy}A_z) + \Omega_{pe}^2(x - v_{\text{ph}}t)A_z = 0, \quad (1)$$

where we have reverted to dimensional units and

$$\Omega_{pe}^2(X) = \frac{4\pi e^2}{m_e} \int_{-\infty}^{+\infty} \frac{f_e(p)dp}{\sqrt{1 + (p/m_e c)^2}}. \quad (2)$$

The last term in the l.h.s. of Eq. (5) is the z -component of the electric current density generated by the electromagnetic wave in a plasma with the electron distribution function $f_e(p)$. In the limit $\gamma_{\text{ph}}\Delta p_0 \ll 1$ for the electromagnetic wave frequency larger than the Langmuir frequency calculated for the maximal electron density, $\omega \gg \omega_{\text{pe}}(2\gamma_{\text{ph}}/\Delta p_0)^{1/4}$, we can neglect the finite temperature effects on the electromagnetic wave dispersion, which have been analyzed in Ref. [13], in the limit of homogeneous, stationary plasmas. For the water-bag distribution function

$$f_e(p, X) = n_0 \theta(p - p_-(X)) \theta(p_+(X) - p) / \Delta p_0 \quad (3)$$

$\Omega_{pe}^2(X)$ takes the form

$$\Omega_{pe}^2(X) = \omega_{pe}^2 \frac{1}{\Delta p_0} \ln \left(\frac{p_+(X) + \sqrt{1 + p_+(X)^2}}{p_-(X) + \sqrt{1 + p_-(X)^2}} \right), \quad (4)$$

where $\omega_{pe}^2 = 4\pi n_0 e^2 / m_e$, $p_{\pm}(X)$ and Δp_0 are now dimensionless (normalized on $m_e c$).

The wake wave modulates the electron density and temperature increasing them in the compression regions and decreasing them in the rarefaction regions. In Fig. 1 we illustrate the dependence of $\Omega_{pe}(X)/\omega_{pe}$ on X for the parameters of a wakewave corresponding to $\Delta p_0 = 0.1$ and $E_{\max} = 2.3$ at $X = 15$ and for $\beta_{ph} = 0.992$.

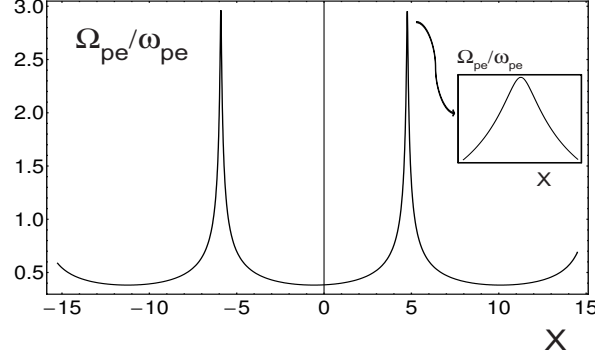


FIG. 1. Dependence of the frequency ratio Ω_{pe}/ω_{pe} on the coordinate X for the parameters of a wakewave corresponding to $\Delta p_0 = 0.1$ and $E_{\max} = 2.3$ at $X = 15$ and for $\beta_{ph} = 0.992$. In the inset the ratio $\Omega_{pe}(X)/\omega_{pe}$ is shown in the vicinity of the maximum.

From Eqs. (35) and (59) of Part I [1] in the ultrarelativistic case, $\beta_{ph} \approx 1$, using Eq. (4) we find for a relatively cold distribution such that $p_- \ll 1$ that near the wavebreaking point $\Omega_{pe}^2(X)$ is given by

$$\Omega_{pe}^2(X) \approx \frac{\omega_{pe}^2}{\gamma_{ph}} - \frac{\omega_{pe}^2 \sqrt{n_{br} \gamma_{ph}}}{\Delta p_0} |X|. \quad (5)$$

The propagation of a sufficiently short electromagnetic wave packet in the plasma with electron density modulated by the Langmuir wave can be described within the framework of the geometric optics approximation. The electromagnetic wave is represented as a particle (“photon”) with coordinate x and momentum \mathbf{k} (wave vector). The interaction of a “photon” with a Langmuir wave that propagates with a relativistic phase velocity $v_{ph} \approx c$ can be accompanied by a substantial frequency upshift called “photon acceleration” [14–17]. Using the dispersion equation

$$\omega(x, \mathbf{k}; t) = \sqrt{k^2 c^2 + \Omega_{pe}^2(x - v_{ph} t)}, \quad (6)$$

where $k^2 = k_{||}^2 + k_{\perp}^2$ with $k_{||}$ and k_{\perp} the wave vector components parallel and perpendicular to the propagation direction of the Langmuir wave, we obtain the “photon” Hamiltonian function which depends on the canonical variables $X = x - v_{ph} t$ and $k_{||}$ (see Ref. [22])

$$\mathcal{H}_{\text{photon}}(X, k_{||}) = \sqrt{k_{||}^2 c^2 + \Omega_{pe}^2(X)} - \beta_{ph} k_{||} c. \quad (7)$$

The transverse component of the wave vector is constant $k_{\perp} = k_{\perp,0}$ and $k_{\perp,0} = 0$ is assumed for the sake of simplicity.

The phase portrait of the photon for $\Omega_{pe}(X)$ given by Eq. (4) for the parameters corresponding to Fig. 1 is shown in Fig. 2.

Along an orbit corresponding to the value $\mathcal{H}_{\text{photon}}(X, k_{||}) = \mathcal{H}_{\text{photon}}(X_0, k_{||,0}) = \mathcal{H}_{\text{photon},0}$ of the Hamiltonian (7) the photon frequency is given by

$$\omega = \gamma_{ph}^2 \mathcal{H}_{\text{photon},0} \left[1 \pm \beta_{ph} \sqrt{1 - \frac{\Omega_{pe}^2(X)}{\mathcal{H}_{\text{photon},0} \gamma_{ph}^2}} \right]. \quad (8)$$

Photons, for which $\mathcal{H}_{\text{photon},0} < \max\{\Omega_{pe}/\gamma_{ph}\}$ are trapped inside the region encircled by the separatrix. Along the orbit their frequency changes from ω_{\max} and ω_{\min} corresponding to the plus and minus signs in the r.h.s. of Eq. (8) at the minimum of $\Omega_{pe}(X)$.

Photons with $\mathcal{H}_{\text{photon},0} > \max\{\Omega_{pe}/\gamma_{ph}\}$ are not trapped and for them the sign in the r.h.s. of Eq. (8) does not change. For trajectories far above the separatrix the photon frequency variations are relatively weak. However a

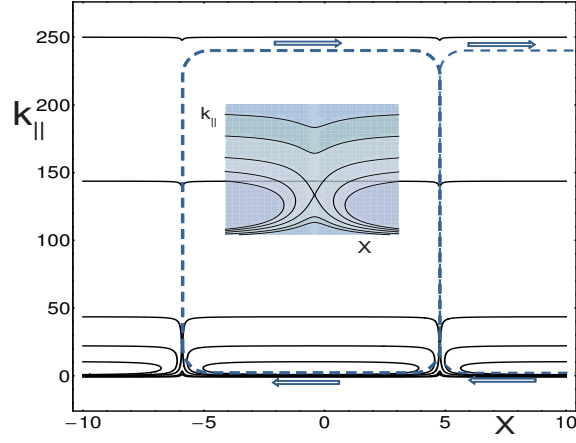


FIG. 2. Photon phase portrait for the parameters of a nonlinear Langmuir wave corresponding to Fig. 1. The dashed line corresponds to the trajectory of photons that have appeared due to the over-barrier reflection at the crest of the breaking wave. In the inset the photon trajectories in the vicinity of the saddle point are shown.

sufficiently strong wakefield can reflect a counterpropagating photon, $k_{||,0} < 0$ due to above-barrier reflection (this trajectory is shown in Fig. 2 by a dashed line). Such a photon acquires a frequency

$$\omega = \omega_0 \frac{1 + \beta_{\text{ph}}}{1 - \beta_{\text{ph}}} \quad (9)$$

according to the Einstein formula for the frequency of the electromagnetic wave reflected by a relativistic mirror [18]. The geometric optics approximation fails when the wakefield is close to wave breaking and this provides the appropriate conditions for a not exponentially weak wave scattering.

III. ABOVE-BARRIER SCATTERING OF AN ELECTROMAGNETIC WAVE AT THE CREST OF THE BREAKING WAKE WAVE

In order to find the reflectivity of the nonlinear wake wave we make a Lorentz transformation to the frame of reference moving with the phase velocity of the Langmuir wave. In the boosted frame, Eq. (5) for the electromagnetic wave interacting with the nonlinear Langmuir wave can be written as

$$\frac{d^2 a(\zeta)}{d\zeta^2} + q^2(\zeta)a(\zeta) = 0 \quad (10)$$

with

$$a(\zeta) = \frac{eA_z}{m_e c^2} \exp[-i(\omega' t' - k_y y)] \quad (11)$$

and in the neighbourhood of the breaking point $q^2(\zeta)$ can be written as

$$q^2(\zeta) = s^2 + g_{-1}|\zeta|. \quad (12)$$

Here

$$s^2 = \frac{\omega'^2}{c^2} - k_y^2 - \frac{\omega_{pe}^2}{c^2 \gamma_{\text{ph}}}, \quad (13)$$

and $\zeta = X\gamma_{\text{ph}}$, t' , k' , ω' are the coordinate and time and the wave number and frequency in the boosted frame of reference. The coefficient g_{-1} is equal to

$$g_{-1} = \frac{\omega_{pe}^2 \sqrt{n_{\text{br}}}}{\Delta p_0 c^2 \sqrt{\gamma_{\text{ph}}}}. \quad (14)$$

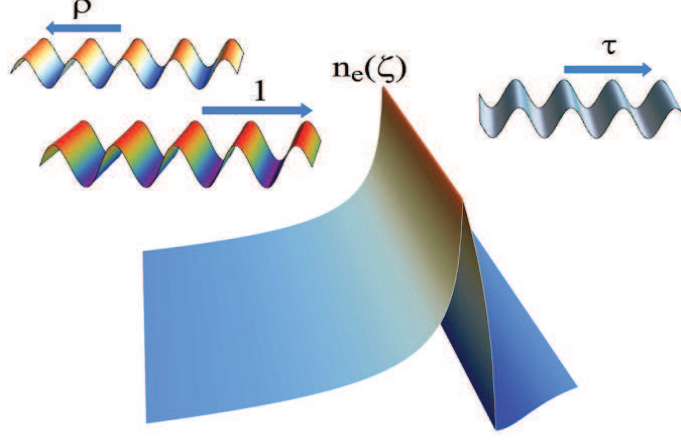


FIG. 3. Scattering geometry.

We seek for the solution to the above-barrier scattering problem for Eq. (10) writing its solution in the form (see Refs. [12, 23, 24])

$$a(\zeta) = \frac{1}{\sqrt{q(\zeta)}} [b_+ \exp(iW(\zeta)) + b_- \exp(-iW(\zeta))], \quad (15)$$

where the phase integral is defined as

$$W(\zeta) = \int_0^\zeta q(\zeta') d\zeta'. \quad (16)$$

The above-barrier scattering geometry is illustrated in Fig. 3. For constant b_+ and b_- Eq. (15) corresponds to the “WKB” solution [25]. In the following the coefficients b_+ and b_- are considered as functions of W instead of ζ , because, as explained in Ref. [21], the mapping between W and ζ given by Eq. (16) is one-to-one on the real axis. Far from the breaking point, i.e. formally for $\zeta \rightarrow \pm\infty$ the function $q^2(\zeta) \rightarrow s^2$ reduces to a constant and the solutions (15) are exact, so that $b_\pm \rightarrow \text{const}$ as $W \rightarrow \pm\infty$.

In other words, the boundary conditions at $\zeta \rightarrow \pm\infty$ are

$$\begin{aligned} b_+(+\infty) &= 1, & b_- (+\infty) &= \rho, \\ b_+(-\infty) &= 0, & b_- (-\infty) &= \tau. \end{aligned} \quad (17)$$

Since in the representation (15), the single unknown function $a(\zeta)$ has been replaced by the two unknown functions $b_\pm(\zeta)$, a subsidiary condition is necessary. We shall impose the condition

$$\frac{da}{d\zeta} = i\sqrt{q(\zeta)} (b_+ e^{iW(\zeta)} - b_- e^{-iW(\zeta)}). \quad (18)$$

Differentiating Eq. (15) with respect to ζ and taking into account the constraint (18), we find

$$\frac{db_+}{d\zeta} e^{iW(\zeta)} + \frac{db_-}{d\zeta} e^{-iW(\zeta)} = \frac{d \ln \sqrt{q}}{d\zeta} (b_+ e^{iW(\zeta)} + b_- e^{-iW(\zeta)}), \quad (19)$$

while differentiating Eq. (18) with respect to ζ and substituting $d^2 a/d\zeta^2$ into Eq. (10) yields

$$\frac{db_+}{d\zeta} e^{iW(\zeta)} - \frac{db_-}{d\zeta} e^{-iW(\zeta)} = \frac{d \ln \sqrt{q}}{d\zeta} (b_- e^{-iW(\zeta)} - b_+ e^{iW(\zeta)}). \quad (20)$$

The system of Eqs. (19) and (20) is equivalent to Eq. (10). It can be rewritten in the form

$$\frac{d}{dW} \begin{pmatrix} b_+ \\ b_- \end{pmatrix} = \begin{pmatrix} 0 & S(W) e^{2iW} \\ S(W) e^{-2iW} & 0 \end{pmatrix} \begin{pmatrix} b_+ \\ b_- \end{pmatrix}, \quad (21)$$

with

$$S(W) = \frac{1}{2} \frac{d}{dW} \ln q(\zeta(W)). \quad (22)$$

For $q(\zeta)$ given by Eq. (12) we have

$$W(\zeta) = \frac{2}{3g_{-1}} \left[(s^2 + g_{-1}|\zeta|)^{3/2} - s^3 \right] \text{sign}(\zeta), \quad (23)$$

where $\text{sign}(\zeta) = -1$ if $\zeta < 0$ and $\text{sign}(\zeta) = 1$ for $\zeta > 0$, and

$$q(W) = \left(\frac{3g_{-1}}{2} W \text{sign}(\zeta) + s^3 \right)^{1/3} \quad (24)$$

so that

$$S(W) = \frac{g_{-1}}{4q^3(\zeta(W))} \text{sign}(\zeta). \quad (25)$$

It follows that $S(W)$ is discontinuous at $\zeta \rightarrow 0$ ($W \rightarrow 0$)

$$S(W=0) = \frac{g_{-1}}{4s^3} \text{sign}(\zeta). \quad (26)$$

Integrating both sides of Eq. (21) and using the above formulated boundary conditions for $b_{\pm}(\pm\infty)$, we can obtain the reflection coefficient ρ in the form of the infinite series [26]

$$\rho = - \sum_{m=0}^{\infty} (-1)^m \int_{-\infty}^{+\infty} dW_0 S(W_0) e^{2iW_0} \prod_{n=1}^m \int_{-\infty}^{W_n-1} dV_n S(V_n) e^{-2iV_n} \int_{V_n}^{+\infty} dW_n S(W_n) e^{2iW_n}, \quad (27)$$

with the product equal to unity for $m = 0$.

The function $q(\zeta)$ defined by Eq. (12) has a discontinuous first derivative at $\zeta = 0$. In the vicinity of the singularity point it can be represented in the form $q(\zeta) \approx q_0 + q_1|\zeta|$ with $q_0 = s$ and $q_1 = g_{-1}/2s$. Expanding $W(\zeta)$ and $S(W)$ in powers of ζ and substituting them into Eq. (27) we can find (see Eq. (27) of Ref. [21]) that the first term yields the dominant contribution to the reflection coefficient, with the result

$$\rho_{-1} \approx \frac{-iq_1}{sq_0} = \frac{-ig_{-1}}{4s^3}. \quad (28)$$

and

$$R_{-1} = |\rho_{-1}|^2 = g_{-1}^2 \frac{1}{s^6}. \quad (29)$$

Applicability of the WKB theory implies that $g_{-1} \ll s^3$.

Similarly (see also Ref. [12]) we can find the reflection coefficient at the electron density singularity formed in the above breaking regime discussed in Part I [1]. In this case the electron density distribution is given by Eq. (106) of Part I. Using this relationship we obtain

$$\rho_{(\frac{1}{2}, \frac{1}{2})} = \frac{-4ig_{(\frac{1}{2}, \frac{1}{2})}}{s} \int_{-\infty}^{+\infty} \exp[2is\zeta] \left(\theta(\zeta) \sqrt{\zeta} - \theta(\zeta - \Delta\zeta) \sqrt{\zeta - \Delta\zeta} \right) d\zeta, \quad (30)$$

where

$$g_{(\frac{1}{2}, \frac{1}{2})} = k_p^{3/2} \gamma_{\text{ph}}^{3/2} \frac{\sqrt{2eE_{\text{max}} m_e c}}{\Delta p_0}, \quad (31)$$

$k_p = c/\omega_{pe}$ and $\Delta\zeta = \Delta p_0/eE_{\text{max}}$. Calculating the integral (30) we find

$$\rho_{(\frac{1}{2}, \frac{1}{2})} = g_{(\frac{1}{2}, \frac{1}{2})} (1+i) \sqrt{2\pi} \frac{\exp(is\Delta\zeta) \sin(s\Delta\zeta)}{s^{5/2}}. \quad (32)$$

Consequently, we write

$$R_{(\frac{1}{2}, \frac{1}{2})} = |\rho_{(\frac{1}{2}, \frac{1}{2})}|^2 = g_{(\frac{1}{2}, \frac{1}{2})}^2 4\pi \frac{\sin^2(s\Delta\zeta)}{s^5}. \quad (33)$$

From Eqs. (29) and (33) we can see that in thermal plasmas the reflection coefficient is $s \gg 1$ times larger in the above breaking regime than for a wake wave approaching the wavebreaking threshold.

Generalizing Eqs. (106) and (108) of Part I [1], we can write the electron density dependence on the coordinate ζ in the form

$$n_e(\zeta) \sim \frac{2n_0}{\Delta\zeta} \left[\theta(\zeta_+) (\zeta_+)^{1/m} - \theta(\zeta_-) (\zeta_-)^{1/m} \right] \quad (34)$$

with $\zeta_{\pm} = \zeta \mp \Delta\zeta/2$ and m an even number, and

$$n_e(\zeta) \sim \frac{n_0}{\Delta\zeta} \left[\theta(\zeta_+) (\zeta_+)^{1/m} + \theta(-\zeta_-) (-\zeta_-)^{1/m} - \theta(\zeta_-) (\zeta_-)^{1/m} - \theta(-\zeta_+) (-\zeta_+)^{1/m} \right] \quad (35)$$

for m an odd number.

It is easy to show that for the reflection coefficient, $R_{(\frac{1}{m}, \frac{1}{m})} = |\rho_{(\frac{1}{m}, \frac{1}{m})}|^2$, we have

$$R_{(\frac{1}{m}, \frac{1}{m})} = g_{(\frac{1}{m}, \frac{1}{m})}^2 \left| 4(-is)^{-1/m} \Gamma\left(1 + \frac{1}{m}\right) \frac{\sin(s\Delta\zeta)}{s\Delta\zeta} \right|^2, \quad (36)$$

if m is even, and

$$R_{(\frac{1}{m}, \frac{1}{m})} = g_{(\frac{1}{m}, \frac{1}{m})}^2 \left| 2(1 + (-1)^{1/m})(-is)^{-1/m} \Gamma\left(1 + \frac{1}{m}\right) \frac{\sin(s\Delta\zeta)}{s\Delta\zeta} \right|^2, \quad (37)$$

if m is odd.

Comparing Eqs (33, 36, 37) for the reflection coefficient with the corresponding coefficients obtained in Ref. [12], we find that the effects of a finite temperature enter Eqs. (33, 36, 37) as a form-factor $|\sin(s\Delta\zeta)/s\Delta\zeta|^2$. In the limit $\Delta p_0 \rightarrow 0$ this form factor tends to unity while for $s\Delta\zeta \gg 1$ decreases as $\approx 1/(s\Delta\zeta)^2$.

Since the frequency, ω_r , and the number of reflected photons, N_r , are related to that incident on the relativistic mirror ω_0 and N_S as $\omega_r = \omega_0(1 + \beta_{ph})/(1 - \beta_{ph}) \approx \omega_0 4\gamma_{ph}^2$ and $N_r = RN_S$, the energy of the reflected photon beam is given by $\mathcal{E}_r \approx \mathcal{E}_S 4\gamma_{ph}^2 R$, where \mathcal{E}_S is the energy of the laser pulse incident on the mirror. Comparing \mathcal{E}_r with the energy of the electrons in the first period of the wake wave (e.g. see [28]), $\mathcal{E}_e \approx \mathcal{E}_{las,d}(\omega_{pe}/\omega_0)^2$, where $\mathcal{E}_{las,d}$ is the laser driver energy, we find that the photon back reaction (the ponderomotive pressure) on the wake wave can be neglected provided $\mathcal{E}_S \ll \mathcal{E}_{las,d}/4\gamma_{ph}^4 R$. As a typical reflection coefficient value we can take $R \approx 1/\gamma_{ph}^4$ (see Refs.[9, 12]) and obtain the condition of relative weakness of the incident laser pulse $\mathcal{E}_S \leq \mathcal{E}_{las,d}$. As we see owing to the weakness of the photon-wake wave interaction the incident pulse energy can be of the order of that in the driver laser pulse.

IV. DISCUSSIONS AND CONCLUSIONS

In the first Part of our paper [1] we found the structure of the typical singularities that appear in the electron density during the wave breaking in a thermal plasma. The singularity in the electron density, moving along with the wake wave excited by a high intensity ultra-short pulse laser, can act as a flying relativistic mirror for counterpropagating electromagnetic radiation, leading to coherent reflection accompanied by the upshift of the radiation frequency. This process implies finite (not exponentially small) reflectivity at the electron density singularities. This is provided by the structure of the singularity formed in a relativistically large amplitude plasma wave close to the wavebreaking limit that leads to a refraction coefficient with discontinuous coordinate derivatives. We found the reflection coefficients of an electromagnetic wave at the singularities of the electron density in the most typical regimes of strongly nonlinear wave breaking in thermal plasmas. The efficiency of the photon reflection can be substantially increased by using the above breaking limit regimes which lead to the formation of high-order singularities.

ACKNOWLEDGMENTS

We acknowledge support of this work from We acknowledge the support from the MEXT of Japan, Grant-in-Aid for Scientific Research, 23740413, and Grant-in-Aid for Young Scientists 21740302 from MEXT. We appreciate support from the NSF under Grant No. PHY-0935197 and the Office of Science of the US DOE under Contract No. DE-AC02-05CH11231.

-
- [1] S. V. Bulanov, T. Zh. Esirkepov, M. Kando, J. K. Koga, A. S. Pirozhkov, T. Nakamura, S. S. Bulanov, C. B. Schroeder, E. Esarey, F. Califano, and F. Pegoraro, *Phys. Plasmas* (2012) - submitted for publication; [arXiv e-print: 2012ArXiv1202.1903B].
 - [2] R. C. Davidson, *Methods in nonlinear plasma theory* (Academic Press Inc., New York, 1972).
 - [3] G. G. Stokes, *Trans. Cambridge Philos. Soc.* **8**, 441 (1847); G. G. Stokes, *Mathematical and physical papers*, vol. I, pp. 197-219, (Cambridge, 1880); J. Wilkening, *Phys. Rev. Lett.* **107**, 184501 (2011).
 - [4] G. B. Whitham, *Linear and Nonlinear Waves* (Wiley-Interscience, New York, 1974).
 - [5] R. Camassa and D. D. Holm, *Phys. Rev. Lett.* **71**, 1661 (1993); A. Degasperis and M. Procesi, in: *Symmetry and Perturbation Theory*// eds. A. Degasperis and G. Gaeta (River Edge, NJ: World Scientific, 1999), pp. 23-37.
 - [6] S. V. Bulanov, I. N. Inovenkov, V. I. Kirsanov, N. M. Naumova, A. S. Sakharov, *Sov. Phys. Lebedev. Inst. Rep.* **6**, 9 (1991) [*Kratk. Soobshch. Fiz.* **6**, 9 (1991)]; S. V. Bulanov, F. Califano, G. I. Dudnikova, T. Zh. Esirkepov, I. N. Inovenkov, F. F. Kamenets, T. V. Liseikina, M. Lontano, K. Mima, N. M. Naumova, K. Nishihara, F. Pegoraro, H. Ruhl, A. S. Sakharov, Y. Sentoku, V. A. Vshivkov, V. V. Zhakhovskii, *Reviews of Plasma Physics*, edited by V. D. Shafranov (Kluwer Academic/Plenum, New York, 2001), Vol. 22, p. 227.
 - [7] S. V. Bulanov, T. Zh. Esirkepov, and T. Tajima, *Phys. Rev. Lett.* **91**, 085001 (2003).
 - [8] M. Kando, Y. Fukuda, A. S. Pirozhkov, J. Ma, I. Daito, L.-M. Chen, T. Zh. Esirkepov, K. Ogura, T. Homma, Y. Hayashi, H. Kotaki, A. Sagisaka, M. Mori, J. K. Koga, H. Daido, S.V. Bulanov, T. Kimura, Y. Kato, and T. Tajima, *Phys. Rev. Lett.* **99**, 135001 (2007).
 - [9] A. S. Pirozhkov, J. Ma, M. Kando, T. Zh. Esirkepov, Y. Fukuda, L.-M. Chen, I. Daito, K. Ogura, T. Homma, Y. Hayashi, H. Kotaki, A. Sagisaka, M. Mori, J. K. Koga, T. Kawachi, H. Daido, S. V. Bulanov, T. Kimura, Y. Kato, and T. Tajima, *Phys. Plasmas* **14**, 123106 (2007).
 - [10] M. Kando, A. S. Pirozhkov, K. Kawase, T. Zh. Esirkepov, Y. Fukuda, H. Kiriyaama, H. Okada, I. Daito, T. Kameshima, Y. Hayashi, H. Kotaki, M. Mori, J. K. Koga, H. Daido, A. Ya. Faenov, T. Pikuz, J. Ma, L.-M. Chen, E. N. Ragozin, T. Kawachi, Y. Kato, T. Tajima, and S. V. Bulanov, *Phys. Rev. Lett.* **103**, 235003 (2009).
 - [11] S. S. Bulanov, T. Zh. Esirkepov, F. F. Kamenets, F. Pegoraro, *Phys. Rev. E* **73**, 036408 (2006); S. S. Bulanov, A. Maximchuk, C. B. Schroeder, A. G. Zhidkov, E. Esarey, W.P. Leemans, *Phys. Plasmas* **19**, 020702 (2012).
 - [12] A. V. Panchenko, T. Zh. Esirkepov, A. S. Pirozhkov, M. Kando, F. F. Kamenets, and S. V. Bulanov, *Phys. Rev. E* **78**, 056402 (2008).
 - [13] V. P. Silin, *Sov. Phys. JETP* **11**, 1136 (1960); B. Kurşunoğlu, *Nuclear Fusion* **1**, 213 (1961); A. B. Mikhajlovskii, *Plasma Phys.* **22**, 133 (1980); D. B. Melrose, *Aust. J. Phys.* **35**, 41 (1982); J. Bergman and B. Eliasson, *Phys. Plasmas* **8**, 1482 (2001).
 - [14] S. C. Wilks, J. M. Dawson, W. B. Mori, T. Katsouleas, and M. E. Jones, *Phys. Rev. Lett.* **62**, 2600 (1989).
 - [15] J. T. Mendonca, *Photon Acceleration in Plasmas* (IOP, Bristol, 2001).
 - [16] V. A. Mironov, A. M. Sergeev, E. V. Vanin and G. Brodin, *Phys. Rev. A* **42**, 4862 (1990); S. V. Bulanov and A. S. Sakharov, *JETP Lett.* **54**, 203 (1991); V. A. Mironov, A. M. Sergeev, E. V. Vanin, G. Brodin and J. Lundberg, *Phys. Rev. A* **46**, R6178 (1992); J. T. Mendonca and L. O. Silva, *Phys. Rev. E* **49**, 3520 (1994); R. Bingham, J. T. Mendonca, J. M. Dawson, *Phys. Rev. Lett.* **78**, 247 (1997); L. O. Silva and J. T. Mendonca, *Phys. Rev. E* **57**, 3423 (1998); A. A. Solodov, P. Mora, and P. Chessa, *Phys. Plasmas* **6**, 503 (1999); A. Spitkovsky and P. Chen, *Phys. Lett. A* **296**, 125 (2002); G. Raj, M. R. Islam, B. Ersfeld, and D. A. Jaroszynski, *Phys. Plasmas* **17**, 073102 (2010).
 - [17] C. W. Siders, S. P. LeBlanc, D. Fisher, T. Tajima, M. C. Downer, A. Babine, A. Stepanov, and A. Sergeev, *Phys. Rev. Lett.* **76**, 3570 (1996); J. M. Dias, C. Stenz, N. Lopes, X. Badiche, F. Blasco, A. Dos Santos, L. O. Silva, A. Mysyrowicz, A. Antonetti, and J. T. Mendonca, *Phys. Rev. Lett.* **78**, 4773 (1997); C. D. Murphy, R. Trines, J. Vieira, A. J. W. Reitsma, R. Bingham, J. L. Collier, E. J. Divall, P. S. Foster, C. J. Hooker, A. J. Langley, P. A. Norreys, R. A. Fonseca, F. Fiuza, L. O. Silva, J. T. Mendonca, W. B. Mori, J. G. Gallacher, R. Viskup, D. A. Jaroszynski, S. P. D. Mangles, A. G. R. Thomas, K. Krushelnick, and Z. Najmudin, *Phys. Plasmas* **13**, 033108 (2006); R. M. G. M. Trines, C. D. Murphy, K. L. Lancaster, O. Chekhlov, P. A. Norreys, R. Bingham, J. T. Mendonca, L. O. Silva, S. P. D. Mangles, C. Kamperidis, A. Thomas, K. Krushelnick, and Z. Najmudin, *Plasma Phys. Control. Fusion* **51**, 024008 (2009).
 - [18] A. Einstein, *Ann. Phys. (Leipzig)* **17**, 891 (1905).
 - [19] A. I. Akhiezer and R. V. Polovin, *Sov. Phys. JETP* **30**, 915 (1956).
 - [20] A. A. Solodov, V. M. Malkin, and N. J. Fisch, *Phys. Plasmas* **13**, 093102 (2006).
 - [21] M.V. Berry, *J. Phys. A* **15**, 3693 (1982).
 - [22] T. Esirkepov, S. V. Bulanov, M. Yamagiwa, and T. Tajima, *Phys. Rev. Lett.* **96**, 014803 (2006).

- [23] V. L. Pokrovskii, S. K. Savinykh, F. R. Ulinich, *Sov. Phys. JETP* **34**, 879 (1958); V. L. Pokrovskii and I. M. Khalatnikov, *Sov. Phys. JETP* **13**, 1207 (1961).
- [24] J. T. Mendonca and K. Hizanidis, *Plasma Phys. Control. Fusion* **54**, 035006 (2012).
- [25] N. Fröman, P. O. Fröman, JWKB Approximation: Contributions to the theory (North-Holland P.C., Amsterdam, 1965).
- [26] M. V. Berry, K. E. Mount, *Rep. Prog. Phys.* **35**, 315 (1972).
- [27] H. Bremmer, *Commun. Pure Appl. Maths.* **4**, 105 (1951); R. Landauer, *Phys. Rev.* **82**, 80 (1951).
- [28] E. Esarey, C. B. Schroeder, W. P. Leemans, *Rev. Mod. Phys.* **81**, 1229 (2009).